THE NONLINEAR UNIVERSE:
Chaos, Emergence, Life

Alwyn Scott

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Recreation of Scott Russell’s soliton on Scotland’s Union Canal, 12 July 1995.
PREFACE

We are all are aware that the twentieth century was an era of unprecedented progress in science and technology. From gravitational dynamics to the fundamental nature of matter, from the physiology of living organisms to the biochemical structures of their constituent molecules, from the evolutionary rules of interaction among biological species and their ever-changing environments to the problems of Earth’s biosphere, from fluid models to numerical predictions of weather and climate, from the dynamics of electromagnetic fields to the designs of the myriad electronic devices with which we are ever becoming familiar, our perspectives are far broader and deeper than they were a century ago. Yet science remains an exciting and unfinished story.

Some of the outstanding questions are of a practical nature and lie in the line of present progress. Can we observe gravitational waves and study their dynamical nature? What is the quantum theory trying to tell us? Is the geometrical theory of gravity related to quantum theory? How did living organisms manage to emerge from the hot, chemical soup of Earth’s Hadean eon, some four billion years ago? Where is Life headed? Will the future of our planet be devastated by the currently massive release of bound carbon atoms to atmospheric carbon dioxide? To what new vistas are current developments in the storage, transmission and manipulation of information leading us?

Other problems are of a more philosophical nature, pushing the envelope of current scientific understanding. Are predictions of all future events from present knowledge possible “in principle” – as some scientists continue to believe – or are important aspects of dynamics chaotic, precluding this possibility? What is the relationship between quantum theory and chaos? Where does the widely-noted “butterfly effect” leave the concept of causality? Indeed, what do we mean by causality? What is the fundamental nature of emergence, when qualitatively new entities come into being? Is there “something new under the sun”? Does emergence lead to new things or merely epiphenomena? How is the phenomenon of biological evolution to be viewed? Are chaos and emergence related? Can we comprehend Life?

Although several practical questions are considered, this book is primarily concerned with the latter class of more general questions. To this end, it describes a revolutionary advance in scientific thinking that occurred during the seventies and eighties of the past century: a Kuhnian development of new
paradigms of research in nonlinear science, which will have increasing importance during the present century. Some precursors of this revolution could be discerned in the 1960s, but nonlinear science is now established as a vigorous intellectual activity, world-wide in scope and suggesting the need for reappraisals of our collective understandings of the natures of chaos and emergence and of the relationship between them.

Far from esoteric, nonlinear science has many applications, including understanding the structures of the elementary particles of matter and the dynamics of the biological molecules upon which Life is based, predicting and retrodicting details of planetary motions and galactic dynamics, sorting out the oft-conflicting claims of zoological and psychological theories that stem from Darwin’s theory of evolution, appreciating both the possibilities and the limitations of weather prediction, coming to terms with global warming, accepting the reality of the widely-reported and long-denied existence of “rogue waves” on the surface of the ocean and appreciating the implications for naval architecture, and becoming aware of how cultural patterns of behavior develop and influence human conduct, among other important examples. Beyond describing these direct applications of nonlinear science, this book goes on to suggest how a science of Life will ultimately be formulated.

Written for general readers who would understand science and for university undergraduates who would become researchers in or teachers of science, the book begins with qualitative descriptions of the three fundamental facets of nonlinear science: chaos, the emergence of independent entities in energy-conserving systems, and the qualitatively quite different emergence of independent entities in dissipative (nonconservative) systems. Like the legs of a milk stool, these three facets are interrelated, with important transitional systems that carry analyses smoothly from one limiting formulation to another as appropriate parameters are varied. Having established the basic nature of nonlinear science and sketched some general approaches to studies of nonlinear systems, backbone chapters present applications to the physical and biological sciences. For describing even more complicated phenomena – fluid turbulence, biologic evolution, and the dynamics of Life, for examples – the more general concept of chaotic emergence is useful.

While remaining grounded in the physical nature of reality, the book closes with a critical examination of the reductionism, the belief that all of dynamics can be reduced to formulations based on interactions among lower level entities (atoms, molecules, genetic codes, neurons, or whatever). A proper appraisal of this widespread belief is needed for a realistic science of Life.

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Chapter 1

Introduction

I just want to know what Truth is!
Thomas Kuhn

In addition to providing us with the above epigraph [Kuhn00],\(^1\) Thomas Kuhn wrote a book on the history and philosophy of science entitled *The Structure of Scientific Revolutions*\(^2\) [Kuhn96], which has sold over a million copies and remains in print after four and a half decades. Upon first reading this book in the early 1960s, I did not realize that the program of research that I was then embarking upon – theoretical and experimental studies of nonlinear wave motion – would soon become part of a Kuhnian revolution.

In *Structure* (as he called his book), Kuhn famously proposed that the history of science comprises two qualitatively different types of activity. First are the eras of “normal science,” during which the widely-accepted models of collective understanding – he called them “paradigms” – are agreed upon, and the primary activity is “puzzle solving” by adepts of those paradigms [Kuhn96]. Occasionally, these eras of normal science are “punctuated” by “revolutionary periods” of varying magnitude and theoretical importance, during which the previously accepted paradigms change rapidly, leading the scientific community to see the world in new ways. According to Kuhn, such rapid changes in scientific perspective occur in a “Gestalt-like manner,” qualitatively similar to the psychological changes of perception where one sees a familiar image in a new way, and also with the rapid development of a new species in the “punctuated evolution” of NeoDarwinism [Mayr02]. After a collective switch of perceptions, the “lexicon” that scientists use to describe reality changes – new words are coined for new concepts, and old words are assigned new meanings, often with implications for the wider culture.

An example of this phenomenon that was studied in detail by Kuhn in the 1950s is the Copernican Revolution [Kuhn57], during which the European per-

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\(^{1}\)See the Reference section for these citations.

\(^{2}\)The third edition of Kuhn’s classic book is particularly valuable as it includes a final chapter responding to critics of the first edition.
ception of planetary motions changed from the geocentric (Ptolemaic) picture to the heliocentric view proposed in 1543 by Copernicus in his *De Revolutionibus Orbium Caelestium*. Although the heliocentric idea had been suggested by Aristarchus of Samos in the third century BCE and seems evident to us now – familiar as we are with the amazing photographs that space probes have sent back from remote corners of the solar system – heliocentrism was not obvious to astronomers of the sixteenth century. In addition to the fact that Earth does not *seem* to move, Ptolemy’s analytic formulation of Aristotle’s cosmology predicted all observations of celestial motions recorded over many centuries by Chinese, Greek, Islamic and European astronomers, and a geocentric Universe is in accord with the Christian myths that had recently been so vividly described by Dante Alighieri in his classic *La Divina Commedia*.

The basic structure of the Ptolemaic Universe consisted of two primary spheres, the inner one being the surface of the earth and the outer (stellar) sphere carrying the stars around us, once every day. Within the stellar sphere were seven lesser spheres – the “orbium” of Copernicus’s title – inhabited by the seven moving bodies: Saturn, Jupiter, Mars, Sun, Mercury, Venus, and Moon, all of which were driven in their motions by the daily rotation of the outer (stellar) sphere. Beyond the stellar sphere there was nothing, as the Ptolemaic Universe was finite.

Because a daily rotation of the stars about a fixed Earth is the simplest way to think about some calculations, the geocentric formulation is still used to teach celestial navigation, and although we modern scientists snicker at the absurd notions of astrology – which claims that motions of the heavenly bodies can influence terrestrial events – this concept is not unreasonable in the context of Ptolemaic astronomy. Aristotle supposed that the fixed motion of the stars causes Saturn’s motion, which in turn causes Jupiter’s motion, and so on, suggesting that some terrestrial phenomena could be partially influenced by the motions of the stars and planets, and in accord with this view, it is well established that the ocean tides are governed by motions of the Sun and Moon. That we now find such difficulty seeing the Universe through Ptolemaic eyes shows not how dimwitted our ancestors were but how markedly our collective view of reality has been changed. And as the polemics over Kuhn’s work have shown, it is difficult for some modern thinkers to accept that Ptolemaic truth seemed as valid as ours in its day [Kuhn96].

Why then did Copernicus propose a heliocentric formulation of astronomy? He had long been concerned with a “problem of the planets” – when Mercury, Venus, Mars, Jupiter and Saturn appear brightest, their motions through the heavens cease and reverse directions for a while before resuming their more regular westward paths. Although the Ptolemaic astronomers could describe and predict these retrograde motions through a well-defined system of deferents, epicycles, ecliptics and equants, there is an *ad hoc* character of their explanations that can be avoided by assuming Earth to be a planet lying between Venus and Mars, rotating on its polar axis each day, and revolving around the Sun. Although the Ptolemaics had a credible explanation for this retrograde motion, Copernicus did not believe that they had the *correct* explanation.
Within a century of Copernicus’s death, Johannes Kepler used the careful observations of his colleague Tycho Brahe – the best pre-telescopic data then available – to show that planets, including Earth, can be more simply and accurately assumed to follow elliptical orbits about the Sun. Furthermore, he observed that a line joining a planet to the Sun sweeps out equal areas in equal times, a result that became known as “Kepler’s law.” Galileo Galilei’s application of the newly-invented telescope to celestial observations then revealed the moons of Jupiter and the phases of Venus, adding further data in support of the heliocentric theory.

In addition to the religious implications of Earth no longer being located at the center of the Universe, the Copernican revolution also altered the scientific concept of motion. Aristotle’s fundamental picture was that moving objects are impelled to move toward or away from the center of the Universe, whereas the corresponding Galilean picture was of isolated massive bodies ideally moving in an infinite Universe with constant speed along straight lines. Galileo assumed that such uniform motion would continue until and unless a mass is acted upon by gravity, impact, friction, or other mechanical forces. Thus the stage was set for Isaac Newton – born just a century after the death of Copernicus and within a year of Galileo’s death – to propose a self-consistent dynamical model of the Universe in his *Principia Mathematica* (entitled to complement René Descartes *Principia Philosophiae*). Being from Britain, Newton was truly standing “on the shoulders of giants” [Hawking04], as the work leading to his *Principia* comprised the efforts of an impressive international group, involving essential contributions from Greece (Aristotle and Ptolemy), Poland (Copernicus), Denmark (Brahe), Germany (Kepler), Italy (Galileo), and France (Descartes) – all member states of modern Europe.

My central claim in this book is that the concepts of nonlinear science comprise a Kuhnian revolution which will have profound implications for scientific research in the present century. As we shall see, research in nonlinear science underwent significant changes over the last three decades of the twentieth century, particularly during the 1970s. Before this decade, important ideas lay undiscovered or were not widely noted, and communications among researchers doing mathematically related work in different fields of science varied from poor to nonexistent. Nowadays, these conditions have changed dramatically. Several international conferences on nonlinear science are held every year, mixing participants from a variety of professional backgrounds to a degree that was not imagined in the 1960s. Nonlinear science centers have spread across the globe, bringing together diversely educated young researchers to collaborate on interdisciplinary activities, combining their skills in unexpected ways. Dozens of nonlinear science journals have been launched, and hosts of textbooks and monographs are now available for introductory and advanced courses in nonlinear science.

In addition to its many interesting and important applications in the physical sciences and technology, we shall see that nonlinear science offers new perspectives on biology and answers a deep question that arose in the context of Newton’s mechanistic model of the Universe: What is the nature of Life?
1.1 What Is Nonlinear Science?

When asked this question at a cocktail party, I often paraphrase Aristotle, saying that nonlinear science is the study of those dynamic phenomena for which the whole differs from the sum of its parts [24] – or just claim it is the science of Life. In other words, particular effects cannot be assigned to particular causal components (as is so for linear systems) because all components interact with each other. If she does not disappear to refresh her drink, I proceed by pointing to dynamic phenomena in virtually every area of modern research that are currently being investigated under the aegis of nonlinear science, including the following:

- **Chaos** (sensitive dependence on initial conditions or the “butterfly effect,” strange attractors, Julia and Mandelbrot sets, problematic aspects of weather prediction, executive toys, electronic circuits),
- **Turbulence** (wakes of ships, aircraft and bullets; waterfalls; clear air turbulence; breaking waves; fibrillation dynamics of heart muscle),
- **Emergent structures** (chemical molecules, planets, tornadoes, tsunamis, rogue waves, lynch mobs, optical solitons, black holes, flocks of birds and schools of fish, cities, Jupiter’s Great Red Spot, nerve impulses),
- **Filamentation** (rivers, bolts of lightning, woodland paths, optical filaments, rain dripping down window panes),
- **Threshold phenomena** (an electric wall switch, the trigger of a pistol, electronic flip-flop circuits, tipping points, the all-or-nothing behavior of a neuron),
- **Spontaneous pattern formation** (fairy rings of mushrooms, the Gulf Stream, ecological domains, biological morphogenesis),
- **Phase changes** (freezing and boiling of liquids, the onset of superconductivity in low temperature metals, superfluidity in liquid helium, magnetization in ferromagnetic materials, polarization in ferroelectric materials),
- **Harmonic generation** (digital tuning of radio receivers, conversion of laser light from red to blue, symphonic music and overdriven amplifiers for rock bands),
- **Synchronization** (coupling of pendulum clocks, mutual entrainment of electric power generators connected to a common grid, circadian rhythms, hibernation of bears, coordinated flashing of Indonesian fireflies),
- **Shock waves** (sonic booms of jet airplanes, the sound of a cannon, bow waves of a boat, sudden pileups in smoothly-flowing automobile traffic),
- **Hierarchical systems** (stock markets, the World Wide Web, economies, cities, living organisms, human cultures),
1.1. WHAT IS NONLINEAR SCIENCE?

- Psychological phenomena (Gestalt perceptions, anger, depression, startle reflex, love, hate, ideation), and
- Social phenomena (lynch mobs, war hysteria, emergence of cultural patterns, development of natural languages).

All of these phenomena and more comprise the subject matter of nonlinear science, which is in some sense a metascience with roots reaching into widely diverse areas of modern research.3

In the United States, the first use of the term “nonlinear science” may have been in a 1977 letter written by Joseph Ford to his colleagues, which defined our subject and is included here as the epigraph to Chap. 5 [740].4 This letter was historically important as it introduced Ford’s *Nonlinear Science Abstracts*, an ambitious project that soon evolved into *Physica D: Nonlinear Phenomena* – the first journal devoted to nonlinear science. Since the middle of the twentieth century, of course, the adjective “nonlinear” has been employed to modify such nouns as: analysis, dynamics, mechanics, oscillations, problems, research, systems, theory, and waves – particularly in the Soviet Union [80, 632] – but Ford defined a broad and cohesive field of interrelated activities; thus it is his sense of the term “nonlinear science” that is used in this book.

A yet deeper characterization of nonlinear science recognizes that the definition of nonlinearity involves assumptions about the nature of causality. Interestingly, the concept of causality was carefully discussed by Aristotle some twenty-three centuries ago in his *Physics*, where it is asserted that [23]:

> We have to consider in how many senses *because* may answer the question *why*.

As a “rough classification of the causal determinants of things,” Aristotle went on to suggest four types of cause [119].

- **Material cause.** Material cause stems from the presence of some physical substance that is needed for a particular outcome. Following Aristotle’s suggestion that bronze is an essential factor in the making of a bronze statue, many other examples come to mind: atoms of iron are necessary to produce hemoglobin, obesity in the United States is materially caused by our overproduction of corn, water is essential for Life. At a particular level of description, a material cause may be considered as a time or space average over dynamic variables at lower levels of description, entering as a slowly varying parameter at a higher level of interest.

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3A fairly complete listing and description of such applications can be found in the recently published *Encyclopedia of Nonlinear Science*, which aims to make the facets of the field available to students at the undergraduate level [741].

4Joseph Ford (1927–1995) was both fun to be around and an inspiration to many in the early years of research in nonlinear science. Ever striving to understand the philosophical implications of chaos, Joe was often at odds with the physics community, but without his research and his encouragement of others, the revolution described in this chapter would have had an even more difficult birth.
CHAPTER 1. INTRODUCTION

- **FORMAL CAUSE.** For some particular outcome to occur, the requisite materials must be arranged in an appropriate form. The blueprints of a house are necessary for its construction, the DNA sequence of a gene is required for synthesis of the corresponding protein, and a pianist needs the score to play a concerto. At a particular level of description, formal causes might arise from the more slowly varying values of dynamic variables at higher levels, which then enter as boundary conditions at the level of interest.

- **EFFICIENT CAUSE.** For something to happen, there must be an “agent that produces the effect and starts the material on its way.” Thus, a golf ball moves through the air along a certain trajectory because it was struck at a particular instant of time by the head of a properly swung club. Similarly, a radio wave is launched in response to the alternating current that is forced to flow through an antenna. Following Galileo, this is the limited sense in which physical scientists now use the term causality [119]; thus an efficient cause is usually represented by a stimulation-response relationship, which can be formulated as a differential equation with a dependent variable that responds to a forcing term.

- **FINAL CAUSE.** Events may come about because they are desired by some intentional organism. Thus a house is built – involving the assembly of materials, reading of plans, sawing of wood, and pounding of nails – because someone wishes to have shelter from the elements, and economic transactions are motivated by future expectations [397]. Purposive answers to the question “why?” seem problematic in the biological sciences, and they emerge as central issues in the social sciences because such phenomena don’t conform to a general belief in reductionism [736, 739]. As we will see in Chap. 8, final causes offer additional means for closed causal loops of dynamic activity which must be included in realistic models, leading to a class of physical systems that cannot be simulated [689].

In more modern (if not more precise) terms, Aristotle’s material and formal causes are sometimes grouped together as distal causes, his efficient cause is called a proximal cause, and his formal cause is either disparaged as a teleological cause or disregarded altogether. The present-day disdain of many scientists for final causes is a serious oversight, as an event often transpires because some living organism wills it so. The ignoring of such phenomena is rooted in Newtonian reductionism, which we will consider in the closing chapter of this book.

While these classifications may seem tidy, reality is more intricate, as Aristotle was aware [23]. Thus causes may be difficult to sort out in particular cases, with several of them often “coalescing as joint factors in the production of a single effect.” Such interactions among the components of complex causes are a characteristic property of nonlinear phenomena, where distinctions among Aristotle’s “joint factors” are not always easy to make. There is, for example, a subtle difference between formal and efficient causes that appears in the
1.1. WHAT IS NONLINEAR SCIENCE?

metaphor for Norbert Wiener’s cybernetics: the steering mechanism of a ship [874]. If the wheel is connected directly to the rudder (via cables), then the forces exerted by the helmsman’s arms are the efficient cause of the ship’s executing a change of direction. For larger vessels, however, control is established through a servomechanism in which changing the position of the wheel merely resets a pointer that indicates the desired position of the rudder. The forces that move the rudder are generated by a feedback control system (or servomechanism) that minimizes the difference between the actual and desired positions of the rudder. In this case, one might say that the position of the pointer is a formal cause of the ship’s turning, with the servomotor of the control system being the efficient cause. And of course the overall direction of the ship is determined as a final cause by the intentions of the captain and his navigator.

Another example of the difference between formal and efficient causes is provided by the conditions needed to fire the neurons in our brains. If the synaptic weights and threshold are supposed to be constants, they can be viewed as formal causes of a firing event. On a longer time scale associated with learning, however, these parameters change; thus they can be considered collectively as a weight vector that is governed by a learning process and might be classified as efficient causes of neuron ignition [737]. Although the switchings of real neurons are far more intricate than this simple picture suggests, the point remains valid – neural activity is a nonlinear dynamic process, melding many causal factors into the overall outcome.

Finally, when a particular protein molecule is constructed within a living cell, sufficient quantities of appropriate amino acids must be available to the messenger RNA as material causes. The DNA code, determining which amino acids are to be arranged in what order, is a formal cause, and the chemical (electrostatic and valence) forces acting among the constituent atoms are efficient causes. Thus in the realms of the chemical and biological sciences, it is not surprising to find several different types of causes involved in a single nonlinear event – parameter values, boundary conditions, forcing functions, and intentions combining to influence the outcome of a particular dynamics. Can these ideas be extended to social phenomena?

Just as supercooled water, resting quietly in its fluid state, may experience the onset of a phase change during which it suddenly turns to ice, collective social phenomena can unexpectedly arise, sweeping away previous assumptions and introducing new perspectives. Examples of such “social phase changes” abound – the revolutions in eighteenth-century France and twentieth-century Russia, lynch mobs, the outbreak of war, England’s collective heartbreak over the untimely death of Princess Diana, and the Copernican revolution [Kuhn57], among many others. In the 1970s, I claim, something similar happened in the organization and practice of nonlinear science.

Although research in nonlinear dynamics goes back at least to Isaac Newton’s successful treatment of the two-body problem of planetary motion, such activities were until recently scattered among various professional areas, with little awareness of the common mathematical and physical principles involved. Beginning around 1970, this situation changed. Those interested in nonlinear
problems became increasingly aware that dynamic concepts first observed and understood in one field (population biology, for example, or flame-front propagation or nonlinear optics or planetary motion) could be useful in others (such as chemical dynamics or neuroscience or plasma stability or weather prediction). Thus research activities began to be driven more by an interest in generic types of nonlinear phenomena than by specific applications, and the concept of nonlinear science began to emerge. Apart from particular applications, we shall see, there are three broad classes of nonlinear problems.

**LOW-DIMENSIONAL CHAOS.** As discussed in the following chapter, an important discovery of nonlinear science is that one cannot – not even “in principle” – predict the behaviors of certain very simple dynamical systems. Due to a phenomenon now popularly known as the “butterfly effect,” systems with as few as three dependent variables can exhibit “sensitive dependence on initial conditions.” Errors in such systems grow exponentially with time, which renders predictions of future behaviors mathematically impossible beyond a certain characteristic (Lyapunov) time.

**SOLITONS.** In energy-conserving nonlinear fields, it is often observed that energy draws itself together into localized “lumps,” becoming particle-like entities (new “things”) that remain organized in the subsequent course of the dynamics. For an example see the frontispiece, where a hydrodynamic soliton has been generated on a Scottish canal by suddenly stopping a motorboat, whereupon a soliton emerges from the bow wave. Similar examples arise in optics, acoustics, electromagnetics, and theories of elementary particles, among other nonlinear dynamical systems.

**REACTION-DIFFUSION WAVES.** Since the middle of the nineteenth century, it has been known that localized waves of activity travel along nerve fibers, carrying signals along motor nerves to our muscles and from one neuron to another within our brains. As nerve fibers do not conserve energy, their dynamics are characterized by an interplay between the release of stored electrostatic energy and its consumption through dissipative processes (circulating ionic currents). To grasp this phenomenon, think of a candle where chemical energy is stored in the unburned wax and released by a moving flame at the same rate that it is dissipated by radiation of heat and light. Thus a candle models the nonlinear processes on nerve fibers, with the flame corresponding to a nerve impulse – exemplifying a second general type of emergence, distinctly different from that of energy conserving systems.

These three types of nonlinear phenomena – low-dimensional chaos, solitons, and reaction-diffusion fronts – are of central concern in this book. “Chaos” is a familiar word of Greek origin, describing, perhaps correctly, the original character of the Universe, but it is now also used in a new sense to imply...
1.2. AN EXPLOSION OF ACTIVITY

“low-dimensional chaos” in nonlinear science. The term “soliton,” on the other hand, was coined in 1965 by Norman Zabusky and Martin Kruskal to indicate the particle-like properties of the solitary-wave solution of energy-conserving wave systems [906]. Describing processes in which energy (or some other essential quantity) is released by the ongoing dynamics, the adjective “reaction-diffusion” is widely but not universally used; thus such phenomena are also referred to as “self-excited” waves or “self-organizing” waves. Following a coinage by Rem Khokhlov in 1974, they are also called “autowaves” in the Russian literature [582].

![Figure 1.1: The annual number of articles in scientific publications that have used the term CHAOS, SOLITON, and REACTION-DIFFUSION in their titles, abstracts, or key words, and the TOTAL of these three plots. (Data from the Science Citation Index Expanded.) The VERHULST curve is calculated from (1.1) to approximate the TOTAL curve with the parameters given in Table 1.1. (Note that the values of the CHAOS plot are misleadingly high before about 1975, as authors then used the term in its classical meaning.)](image)

1.2 An Explosion of Activity

Although the roots of these three components of modern nonlinear science go back at least to the nineteenth century, the frequency with which they appeared

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6 This new term took a couple of decades to work its way into English dictionaries, and it was not uncommon for manuscripts being published in the 1970s to have “soliton” replaced with “solution” by overly zealous copy editors of physics journals. The word entered the public mind from a Star Trek episode of the early 1990s, and to my great relief it is now in the official Scrabble dictionary.
in scientific publications began to grow explosively 1970, as Fig. 1.1 shows.
More precisely, the curves indicate an exponential rise (or “Gestalt switching”
to use Kuhn’s metaphor) from 1970 to 1990, with a doubling time of about three
years. This was followed by an apparent leveling off (or “saturation”) around
the beginning of the present century at a rate of more than 3000 papers per year,
or about eight per day – evidently a lower estimate because some nonlinear-
science papers don’t use the terms “chaos,” “soliton” or “reaction-diffusion”
in their titles or abstracts. From the perspectives of other manifestations, these
curves look much like the heat emitted from a freshly lit bonfire, the onset of
applause in a theater, or initial the growth of a biological population.

How are we to understand these data? What causes the early rise of the
curves? Why do they saturate? How do they get started? More generally, can
the tools of nonlinear science help us understand a social phenomenon: the
way that modern nonlinear science emerged and grew?

Before introducing a brief mathematical representation of the data in Fig.
1.1, let’s consider qualitative descriptions of the above examples. To get the
dynamics started, some threshold must be overcome: a lighted match for the
bonfire, one person’s burst of enthusiasm in the theater, and the presence of
at least one male and one female of a biological species. Once triggered, these
processes begin to grow as a result of positive feedback around closed causal loops.
Thus the initial burning of the bonfire releases more heat which burns more
fuel which releases even more heat, and so on, in an endless loop of causality.
Similarly, the initial applause in a theater induces more enthusiasm, which
elicits more applause, which induces yet more enthusiasm, etc. In the example
of population growth, the early reproduction rate is proportional to the pop-
ulation size, which again leads to a positive feedback loop and growth at an
exponentially increasing rate. Other examples, albeit with very different time
scales, are the explosive increase of neutrons inside an atomic bomb and the
growth of dandelions on a poorly tended lawn.

To obtain an analytic description of this nonlinear phenomenon, note that
the TOTAL curve in Fig. 1.1 can be closely fitted by the function

$$ N(t) = \frac{N_0 \cdot N(1970) \cdot e^{\lambda(t-1970)}}{N_0 + N(1970) \cdot (e^{\lambda(t-1970)} - 1)}, $$

(1.1)

with the parameters given in Table 1.1. This function was first derived in 1845
by Pierre Francois Verhulst (1804–1849) to represent the growth of biological
populations [840]. He showed that (1.1) is an exact solution to the nonlinear
ordinary differential equation (ODE) [840]

$$ \frac{dN}{dt} = \lambda N \left(1 - \frac{N}{N_0}\right) $$

(1.2)

for the limited growth of a biological population. Equation (1.2) is now widely
known as the “logistic” or “Verhulst” equation, and its solution is one of the
early and exact results from nonlinear science. Impressively, Verhulst used
1.2. AN EXPLOSION OF ACTIVITY

(1.1) to predict the limiting population of his native Belgium to be 9,400,000, whereas the 1994 population was 10,118,000.7

Table 1.1: Fitting parameters in (1.1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting rate:</td>
<td>$N_0$</td>
<td>3100</td>
<td>papers/year</td>
</tr>
<tr>
<td>Initial rate:</td>
<td>$N(1970)$</td>
<td>12</td>
<td>papers/year</td>
</tr>
<tr>
<td>Growth exponent:</td>
<td>$\lambda$</td>
<td>0.25</td>
<td>years$^{-1}$</td>
</tr>
</tbody>
</table>

Additional empirical evidence for an explosion of activity in nonlinear science is provided by the many introductory textbooks8 and advanced monographs9 recently written in the area. Also the histogram in Fig. 1.2 shows an impressive growth in the number of journals committed to publishing research in nonlinear science over recent decades [740]. This is not the whole story, of course, because many of the traditional journals of physics, applied mathematics, theoretical biology, and engineering have carried papers on nonlinear science over the past three decades and continue to do so, but the space in these new journals has contributed largely to the rising publication rate shown in Fig. 1.1, just as publication pressure has led to the new journals.

Another factor associated with the sudden growth of research activity in nonlinear science has been the worldwide emergence of interdisciplinary nonlinear science centers, of which the Center for Studies of Nonlinear Dynamics at the La Jolla Institute (founded in 1978), the Santa Cruz Institute for Nonlinear Science, which grew out of the “Santa Cruz Chaos Collective” in the early 1980s, the Center for Nonlinear Studies at the Los Alamos National Laboratory (founded in 1980), the Institute for Nonlinear Science at UCSD (founded in 1981), and the Santa Fe Institute (founded in 1984) were among the first. There are now dozens of such centers around the globe, dedicated to promoting interrelated studies of chaos and emergent phenomena from the fundamental perspectives of physics, chemistry, mathematics, engineering, biology, psychology, economics, and the social sciences and with emphasis upon particular areas of applied research.

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7To see how these two equations work, note that $e^{\lambda (-1970)} = 1$ at $t = 1970$, so the RHS of (1.1) reduces to $N(1970)$ as it must. For $N(1970) \ll N_0$ at $t = 1970$, both the second term on the RHS of (1.2) and the second term in the denominator of the RHS of (1.1) are small, indicating that the solution, $N(t)$, begins to grow exponentially as $N(t) \approx N(1970)e^{\lambda (-1970)}$. For $e^{\lambda (-1970)} \gg 1$, on the other hand, the RHS of (1.1) is approximately equal to $N_0$, which makes the RHS of (1.2) zero, confirming that $N(t)$ is no longer increasing with time. In other words, the growth has saturated at $N(t) = N_0$.

8See references [2, 161, 178, 218, 221, 415, 422, 452, 456, 461, 462, 519, 604, 676, 738, 828, 872].

Thus three types of data – publication rates of research papers, publications of books and monographs, and the launching of centers for the study of nonlinear science – all suggest that a Kuhnian revolution in this area took place in the latter decades of the twentieth century. But why did this explosion of activity begin in the 1970s? What caused it?

1.3 Causes of the Revolution

Following Aristotle, causes for the above-described explosion of nonlinear science activity can be formal and efficient, where formal causes set the conditions for an explosion to occur, and efficient causes lit the fuse. As is suggested by Edward Lorenz in his book *The Essence of Chaos* [498], a fundamental cause for the growth of nonlinear science research in the 1970s was the recent increase in computing power. Evidence of this dramatic increase is shown in Fig. 1.3, which plots the number of transistors on an Intel processor against time [409]. Between 1970 and 1990, the doubling time of this growth is about two years, which can be compared with Gordon Moore’s 1965 estimate of a one year doubling time (referred to as “Moore’s law” in the public press) [577].

Although many of today’s “computers” are used for other tasks – word processing, record keeping, email, internet searches, and so on – the first electronic computers of the 1950s were designed to calculate solutions to partial differential equations (PDEs), which hitherto had been solved either analytically or with agonizing slowness on mechanical hand calculators. Importantly, it was on one of the very first digital computers – the vacuum-tube MANIAC
at the Los Alamos National Laboratory in the early 1950s – that Enrico Fermi, John Pasta and Stan Ulam carried out the now famous FPU computations, which eventually led Zabusky and Kruskal to their numerical rediscovery of the soliton in the mid-1960s. And numerical studies of mathematical models for weather prediction on an early vacuum-tube computer (the Royal-McBee LPG-30 with a 16 KB memory) in the late 1950s led Lorenz to his unanticipated observation of low-dimensional chaos [497]. In the lexicon of nonlinear science, therefore, the steady increase in computing power shown in Fig. 1.3 is a progressive change in a control parameter, much like the progressive desiccation of a forest as it prepares to burn, the gradual temperature reduction in a beaker of supercooled water, or the rising level of discontent of a rebellious population.

![Figure 1.3: The number of transistors on an Intel processor vs. time (data from [409]).](image)

Yet another formal cause of the explosion of activity shown in Fig. 1.1 has been the accumulation of seemingly unrelated research results on various nonlinear problems over previous decades and centuries, including nineteenth-century research in planetary motions, hydrodynamics, and population growth; mid-twentieth-century investigations of electron beam devices (traveling-wave tubes, and backward-wave oscillators), particle accelerators and plasma confinement machines, nerve-impulse dynamics; and studies of the newly-invented laser and of various types of tunnel diodes in the early 1960s.

In Aristotelian terms, efficient causes of a Gestalt-like switching behavior are pressures that build up and eventually overcome the collective resistance to changes in attitude by practitioners of normal science. According to Kuhn, there are several reasons for resistance to such change. First, of course, most proposals of new theoretical perspectives turn out to be wrong (internally inconsistent, at variance with empirical observations, or both). Second, a new
idea may not be wrong but fail to make new predictions, relegating the choice between competing theories to considerations of analytic convenience or taste. For a century, as we have seen, this was so for the Copernican theory of planetary motions vis-à-vis the Ptolemaic geocentric formulation [Kuhn96], and physicists will recall the competition between the seemingly different quantum theories of Werner Heisenberg and Erwin Schrödinger, which are theoretically equivalent but differ in appearance and ease of application. Third, available texts and university courses limit their content to standard formulations, making it less likely for students to learn of new ideas. Finally, there is the possibility that a new theory is indeed a better representation of reality (more consistent, less ad hoc, in better agreement with experimental data, etc.), but influential leaders have vested interests in the established paradigm, motivating them to construct firm, if irrational, defenses of their traditional positions. An extreme example of such resistance was Galileo’s trial and conviction by the Church of Rome as an heretic for the heliocentric views published in his Dialogue on the Two Principal Systems of the World [Kuhn57]. The objections raised by nineteenth-century religious leaders and also by Harvard’s Louis Agassiz against Charles Darwin’s theory of natural selection as presented in his 1859 book The Origin of Species [182] provide other examples, where, although differently motivated, both religious and scientific leaders favored the previously dominant paradigm of divine creation of immutable biological species [116].

Although studies of nonlinear problems have long been carried on in diverse areas of science, these efforts were largely balkanized, with little awareness of the mathematical principles relating them. Just as progressively supercooled water becomes more and more inclined to freeze (a sudden transition that can be triggered by gently shaking the liquid or dropping in a small crystal of ice), a drying forest becomes more flammable (ready to burn at the careless drop of a match or the strike of a lightning bolt) and a suppressed population becomes more restive (ready to revolt), the scattered results of nonlinear research – particularly during the nineteenth century and the first half of the twentieth century – seems to have reached an unstable level by 1970, leaving the exponential growth shown in Fig. 1.1 ready to emerge. What shook the beaker, lit the match or fired the first shot? How did the exponential growth get started?

A trigger event is also an efficient cause, several of which occurred to ignite the explosion of activity shown in Fig. 1.1. In the summer of 1966, Zabusky and Kruskal organized a NATO-supported International School of Nonlinear Mathematics and Physics at the Max Planck Institute for Physics and Astrophysics in Munich [901]. Following general surveys by Heisenberg on nonlinear problems in physics [372] and by Ulam on nonlinear problems in mathematics, there were focused talks by Nicholas Bloembergen on nonlinear optics [81], Clifford Truesdale on nonlinear field theories in mechanics, John Wheeler on cosmology, Philip Saffman on homogeneous turbulence, Ilya Prigogine on

10Interestingly, some of Agassiz’s arguments are still used to maintain the anti-evolutionary position in benighted subcultures of North America.
nonequilibrium statistical mechanics, and Roald Sagdeev on nonlinear processes in plasmas, among those of several other scientists.

A three-week workshop on Nonlinear Wave Motion was organized in July of 1972 by Alan Newell, Mark Ablowitz and Harvey Segur at Clarkson College of Technology (now Clarkson University) in Potsdam, New York [596]. At this meeting, which was attended by about 60 budding nonlinear scientists of varied backgrounds from several different countries, Pasta described the hitherto perplexing FPU problem [273], and both Kruskal and Peter Lax explained the general structure of the inverse scattering transform (IST), among many other presentations. This IST method had recently been formulated by Kruskal and his colleagues for constructing multi-soliton solutions of the Korteweg-de Vries (KdV) equation, which describes the dynamics of shallow water waves in a one-dimensional channel (see frontispiece) [314, 475]. Making connection with real-world phenomena, Joe Hammack showed that a tsunami can be viewed as a hydrodynamic soliton of the KdV equation [360], and Kruskal, among others, discussed the sine-Gordon (SG) equation, which was known to have an exact two-soliton solution and an infinite number of independent conservation laws. Importantly, the SG equation had previously arisen in the context of discolation dynamics in crystals [299], nonlinear optics [458], Bloch-wall dynamics in magnetic materials [262], and elementary particle theory [643].

One of the most important contributions to Newell’s meeting arrived unexpectedly through the mail. A current copy of Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki appeared, containing an article by Russian scientists Vladimir Zhakarov and Alexey Shabat in which an IST formulation was developed for multi-soliton solutions of the nonlinear Schrödinger (NLS) equation [910], a model that had independently arisen in studies of nonlinear optics both in Russia [622] and in the United states [430] during the mid-1960s. When Hermann Flaschka began to translate this unanticipated contribution at a crowded evening session, a hush fell over the room, and as it became clear that yet another nonlinear dynamical system of practical interest shared the IST properties, excitement among the participants grew. According to my notes from the subsequent discussions, Fred Tappert pointed out that the NLS equation appears in many different applications – including deep water waves [67] and plasma waves [796] in addition to nonlinear optics – because it is generic, arising whenever a wave packet experiences nonlinearity. Robert Miura then spoke about another equation that is closely related to KdV – thus called the “modified KdV” equation or MKdV. Finally, Lax explained how his operator formulation for KdV had been ingeniously applied by Zakharov and Shabat to the NLS equation. Thus many left Newell’s workshop expecting that the special IST properties would generalize to an important class of integrable nonlinear wave equations in which localized lumps of energy are found to emerge. This exciting expectation was soon fulfilled in an important paper by Ablowitz, Newell Segur, and David Kaup, which brought the KdV, MKdV, SG and NLS under the aegis of a unified theoretical picture that included several other nonlinear PDEs [4].
In the following years there were many such workshops, bringing together researchers from diverse academic backgrounds and areas of science. These new relationships and the enlarged perspectives they offered helped to fix nonlinear paradigms in the minds of recently converted acolytes. Communications between Soviet and Western nonlinear scientists were greatly improved by an exciting conference held at the Institute of Theoretical Physics in Kiev in September of 1979 with participants comprising many nonlinear researchers from both sides of the Cold War barrier. During these meetings, many scientific collaborations were begun and friendships formed which significantly influenced the future course of research in nonlinear science.

Also in 1972, a talk entitled “Predictability: Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?” was presented by Lorenz at the annual meeting of the American Association for the Advancement of Science in Washington, D.C. [498]. In other words, Lorenz asked: Is the weather significantly influenced by very small causes? Although similar questions had been raised before, his metaphor for sensitive dependence of a nonlinear system on its initial conditions caught the imagination of the scientific world – not to mention the general public – and studies of chaos in nonlinear dynamical systems of low dimension began to take off.
1.3. CAUSES OF THE REVOLUTION

Lorenz’s 1972 talk was based on an article that he had written for meteorologists a decade earlier [497]. This paper presented results of numerical studies on a highly simplified model of the weather system (only three dynamic variables) which showed that solutions evolving from closely spaced initial conditions would rapidly diverge into very different non-periodic trajectories. Although the implications of Lorenz’s result for weather prediction are now accepted – computer power must grow exponentially in order to achieve a linear increase in prediction time – his paper was largely ignored, accumulating only ten citations (by meteorologists) between 1963 and 1973, less than one per year and an order of magnitude less than the number of papers using the word “chaos” in its traditional sense. After 1975, however, this situation changed dramatically, as is shown by Fig. 1.4. From 1975 to the end of 2003, Lorenz’s paper accumulated a total of about 3300 citations for an average of 114 per year, placing it among the most highly cited publications on nonlinear science.

As the importance of Lorenz’s studies was recognized by the wider scientific community, the term “chaos” came to be used in a new sense, implying sensitive dependence of solution trajectories on their initial conditions and the unanticipated non-periodic behavior of low-dimensional systems that Lorenz had reported a decade earlier. During the rapid rise of interest in deterministic chaos after 1975, amusingly, references to Lorenz’s early paper even exceeded the number of papers using the term “chaos” for several years, reflecting the fact that physical scientists were not yet comfortable with the new definition of this ancient term.

Yet another trigger for the explosive growth of nonlinear science research in the early 1970s was the interest of US applied mathematicians in reaction-diffusion problems. In 1952, the importance of these systems had been established in Britain through experimental studies of nerve impulse propagation by Alan Hodgkin and Andrew Huxley [390]11 and contemporary theoretical work on the problem of biological morphogenesis by Alan Turing [821]. Following these leads, reaction-diffusion systems had been of interest to electrical engineers in several countries, including Denmark, Japan, Russia, and the US, throughout the 1960s. Such problems were taken up by applied mathematicians in the early 1970s after the publication of a highly-visible paper by Henry McKean at New York University [551], which soon led to others [144, 264, 442, 681]. Around this time, Charles Conley – one of the most brilliant and innovative applied mathematicians at the University of Wisconsin – began using reaction diffusion as an example of his geometrical approach to dynamics [166], and he encouraged his students to study reaction diffusion [138, 139, 417].12 Thus by the mid-1970s, research on reaction-diffusion systems was deemed a respectable mathematical activity, leading to many of the publications counted in the lowest curve of Fig. 1.1.

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11In 1963 Hodgkin and Huxley were awarded the Nobel Prize in medicine for this research.
12As I was then working on reaction-diffusion processes in the UW electrical engineering department, Conley drew me into the interdisciplinary applied mathematics community that was gathering at the Mathematics Research Center.
Based on these ideas and events, this book is organized as follows. The next three chapters (2 through 4) give general descriptions of the three components of nonlinear science (chaos, solitons, and reaction-diffusion phenomena), followed by a chapter (5) showing how the components are interrelated and describing general approaches to nonlinear science. Filling out the details of these general chapters are two (6 and 7) on applications of nonlinear concepts to physical problems and to the life sciences. A closing chapter (8) then presents a critical evaluation of reductionism, developing a precise definition of the term “complex system” and suggesting how the realms of nonlinear science may be expanded to include the phenomena of Life.
Chapter 8

Reductionism and Life

There is no record of a successful mathematical theory which would treat the integrated activities of the organism as a whole. [...] And yet this integrated activity of the organism is probably the most essential manifestation of life.
Nicholas Rashevsky

As was emphasized in the first chapter of this book, the nexus between nonlinear science and philosophy can be traced back to Aristotle's four categories of causality [23], because nonlinearity mixes strands of causal implication. Although fully aware of Newtonian physics, modern thinkers often overlook the implications of nonlinear science, tacitly believing instead that chains of logical inference can always be traced from whatever happens back to the sources of what made it so and thereby supporting the devotion to explanatory reductionism that pervades our Western culture. In this chapter, we will see that reductionism ignores the emergent, chaotic and chaotically-emergent phenomena that arise in much of physical science, most of biology and all of the social sciences. In the course of their current attempts to understand the nature of Life, it is hoped that philosophers of science and the literate public will begin to see how complicated reality is, and thus to recognize the implications of the nonlinear-science revolution.

8.1 Newton’s Legacy

Since Aristotle’s mechanics were overturned by Newton’s radical reformulation in the seventeenth century, the reductive program has been successful in prising out explanations for natural phenomena in many fields, in addition to reformulating our collective concept of Nature itself. This program is now widely accepted by the scientific community as the fundamental way to pose and answer questions in a search for truth. How does it go?
8.1.1 The Reductive Program

As most are aware, an effective approach to understanding natural phenomena proceeds in three steps.

- **Analysis.** Assuming some higher-level phenomenon is to be explained, separate the underlying system into disjoint elements, the behaviors of which are to be individually investigated.

- **Theoretical formulation.** Guided by empirical studies, imagination and luck, obtain a formulation (or model) of how the components might be related.

- **Synthesis.** In the context of this model, derive the higher-level phenomenon of interest and quantitatively compare its theoretical behavior with empirical observations.

Studies of natural phenomena that have successfully used this reductive approach include: *planetary motion* (based on the concepts of mass and gravity, related by Newton’s laws of motion), *hydrodynamics* (based on the concepts of fluid density and Newton’s laws, related by the Navier-Stokes equation), *electromagnetic radiation* (based on the concepts of electric charge, electric fields and magnetic fields, related by Maxwell’s electromagnetic equations), *atomic and molecular structures* (based on the concepts of mass and electric charge, related by Schrödinger’s equation for quantum probability amplitudes), *nerve impulse propagation* (based on the concepts of voltage, membrane permeability and ionic current, related by the Hodgkin–Huxley equations for the dynamics of current flow through a voltage sensitive membrane), the *structures of protons and neutrons* (based on the concepts of leptons and quarks and strong, weak, and electromagnetic forces, related by the Standard Model), and *the evolution of our Universe* (based on the concepts of mass and curved space, related by the Einstein–Hilbert gravitational equations) – an impressive list indeed.

Generalizing from such examples, many scientists agree with physicist Steven Weinberg and biologist Jacques Monod that all natural phenomena can be understood “in principle” by reducing them to fundamental laws of physics [575, 864, 865, 867, 868]. Others deny such explanatory reductionism, maintaining that there are natural phenomena that cannot be described in terms of lower-level entities – Life and human consciousness being outstanding examples [19, 169, 540, 541]. In an extreme form, the denial of explanatory reductionism is called *substance dualism*: the view of René Descartes that important aspects of the biological and cognitive realms do not have a physical

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1Weinberg was awarded the 1979 Nobel Prize in Physics for unifying weak and electromagnetic forces.

2In the late 1980s and early 1990s, the polemics between these two camps were sharpened in the United States by a political struggle over an expensive Superconducting Super Collider (SSC), which would test high-energy predictions of the Standard Model. Funding for the SSC was terminated by the House of Representatives in 1993 after two billion dollars had been spent and fourteen miles of tunnel had been dug.
A more moderate position is property dualism (also called functionalism [912]), which accepts a physical basis for Life and Mind but asserts nonetheless that some emergent aspects of the life sciences cannot be explained in terms of atomic or molecular dynamics [281, 540, 912]. How can we decide between these views of Nature?

### 8.1.2 Supervenience and Physicalism

To statements of belief there is not a scientific response, but if we are able to agree on the physical basis of Life and Mind, the scope of the discussion narrows. Let’s assume, therefore, that all biological, cognitive and social phenomena supervene on the physical in the following sense (as philosopher Jaegwon Kim has put it in the context of cognitive science [435, 436]):

Any two things that are exact physical duplicates are exact psychological duplicates as well.

Related to Ernst Mayr’s constitutive reductionism [540, 541], this position is called physicalism by philosophers [912], and among biologists and neuroscientists it is now widely accepted for the phenomena of Life and Mind, whereas Henri Bergson’s concept of a “Life force” (elán vital) [68] is almost universally rejected, not to say despised. Thus two questions arise:

- Does explanatory reductionism follow from physicalism?
- Does physicalism allow property dualism?

Over the past two decades, these questions have been considered by Kim, who reluctantly concludes that physicalism does indeed imply explanatory reductionism and sits uneasily with property dualism [435, 436]. Let us review his arguments with reference to Fig. 8.1.

![Figure 8.1: The causal interaction of higher-level mental phenomena (M₁ and M₂) that supervene on lower-level physical properties (P₁ and P₂).](image)

This figure represents higher-level mental phenomena (M₁ and M₂) that supervene on lower-level physical descriptions (P₁ and P₂), where supervenience is

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3Bergson received the 1927 Nobel Prize in Literature.
indicated by the vertical dashed lines. In other words, there cannot be a difference in \( M_1 \) without a corresponding difference in \( P_1 \), with the same relationship between \( M_2 \) and \( P_2 \) [912].

Now suppose that empirical studies in psychology have established causality between \( M_1 \) and \( M_2 \) (indicated by the horizontal arrow in Fig. 8.1), under which an observation of \( M_1 \) always leads to a subsequent observation of \( M_2 \). Under the assumption of physicalism, \( C_8 \) (\( C_8 \)) must be present to provide a basis for \( C_5 \) (\( C_5 \)), thus we could as well say that \( P_1 \) causes \( P_2 \), which is a formulation of the upper-level causality in terms of the corresponding lower-level properties. In other words, one can reduce the causal relation between \( M_1 \) and \( M_2 \) to a corresponding causal relation between \( P_1 \) and \( P_2 \), thereby supporting explanatory reductionism and undercutting property dualism. There is no claim that such an explanatory reduction is conveniently formulated or has been achieved, but merely that it is possible “in principle.”

8.1.3 Practical Considerations

Beyond Weinberg’s claim and Kim’s logic, there is a practical argument supporting the reductive view. Even if explanatory reductionism were not to hold for all aspects of biological or mental organization, it is still a prudent strategy for biologists and cognitive scientists to take as a working hypothesis, because the riddles of one generation often become standard knowledge of the next. Recalling that physicist Hermann Helmholtz could not come up with an explanation for his measurements of a small speed of nerve impulse propagation [729] and that physiologist Edward Reid’s empirical observations of active ion transport across cell membranes were ignored for seven decades by biologists fearing the taint of Bergsonian vitalism [673, 674, 721], who would claim that a mystery of today couldn’t be reductively explained by a fresh-eyed young thinker of tomorrow? Thus the dualist (substance or property) is ever in danger of giving up too soon, and one might say that it is the duty of a scientist to search for reductive explanations for empirical observations.

Evidently, explanatory reductionism grounded in physicalism is a serious philosophical position. Those who disagree on intuitive grounds, as I do, must offer substantial objections.

8.2 Objections to Reductionism

Although many physicists (especially elementary-particle physicists who seek a “theory of everything”) are reductionists [867, 868], condensed-matter physicists (those who study global phenomena in aggregates of atoms and molecules) often challenge such claims. Thus solid-state physicist Philip Anderson has asserted that [19]:

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4Anderson was awarded the 1977 Nobel Prize in Physics for his work on disordered systems.
the reductionist hypothesis does not by any means imply a “constructionist” one: The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. In fact the more the elementary-particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the very real problems of the rest of science, much less to those of society. The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity.

What is it about “scale and complexity” that creates difficulties for the constructionist hypothesis?

### 8.2.1 Googols of Possibilities

As was discussed in Sec. 7.3.4, biological computations are often problematic because the number of possible emergent structures at each level of Life’s hierarchy (although finite) is too large to be counted.\(^5\) Thus biological science differs fundamentally from physical science, which deals with *homogeneous* sets having identical elements. A physical chemist, for example, has the luxury of performing as many experiments as are needed to establish scientific laws governing the interactions among (say) atoms of carbon and hydrogen as they form identical molecules of benzene. In life sciences, on the other hand, the number of possible members in interesting sets is typically immense, so experiments are necessarily performed on *heterogeneous subsets* of the classes of interest (see Sec. 7.3.5). Because the elements of heterogeneous subsets are never exactly the same, it follows that experiments on them cannot be precisely repeated; therefore causal regularities cannot be determined with the same degree of certainty in the biological, cognitive and social sciences as in the physical sciences.

In other words, life scientists establish *rules* rather than laws for biological and social dynamics, and your doctor can only know the probability that a certain pill will effect a cure. Thus the horizontal arrow from \(M_1\) to \(M_2\) in Fig. 8.1 might be drawn fuzzy or labeled with an estimate of its reliability, in order to indicate a deviation from strict causality.

### 8.2.2 Convoluted Causality

Nonlinear dynamics offer many examples of sensitive dependence on initial conditions, leading to the “fortuitous phenomena” discovered by mathematician Henri Poincaré (see Sec. 2.2 and the epigraph to Chap. 2) and dubbed “the butterfly effect” by meteorologist Edward Lorenz (Sec. 1.3), but such effects have long been informally recognized. Political scientists speak of “tipping

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\(^5\) Such large yet finite numbers – called *immense* by physicist Walter Elsasser [255] – are of the order of a googol \(10^{100}\) or more.
points,” and among computer engineers and neuroscientists, the correspond-
ing idea of a “threshold level” at the input of an information processor (below
and above which different outcomes transpire) is an essential concept [749].

As a problem long of interest to me, consider the dynamics of dendrites,
which carry signals into neurons from sensors and other neurons (see Sec. 7.4.3
and [728, 757, 737]). Until the mid-1980s, it was widely assumed that these
highly branching structures computed linear weighted sums of their inputs,
largely because this perspective helped neuroscientists follow their strands of
theoretical causality. Real dentrites, on the other hand, are now known to be
highly nonlinear, offering many additional tipping points to the dynamics of
every neuron.

How are such twisted skeins of causality to be sorted out? Whether one is
concerned with establishing dynamic laws in the physical sciences or seeking
rules in the life sciences, the notion of causality requires careful considera-
tion [119]. As was noted in Chap. 1, Aristotle identified four types of cause: ma-
terial, formal, efficient, and final [23], and for those familiar with the jargon
of applied mathematics, the following paraphrasing of his definitions may be
helpful.

- At a particular level of the biological hierarchy, a material cause would
  be a time or space average over a dynamic variable at a lower level of
description, which would appear as a slowly varying parameter at the
level of interest. (Examples are the density of atoms in a chemical reaction
or their temperature.)

- Again, at a particular level of the biological hierarchy, a formal cause might
  arise from the more slowly varying values of a dynamic variable at a
higher level of description, which enters as a boundary conditions at the
level of interest. (The walls of a chemist’s test tube, the drum with a
marching column of soldiers, or the temperature inversion inducing the
formation of a tornado are examples.)

- An efficient cause is represented by a stimulation-response relationship, which
  is usually formulated as a differential equation with a dependent vari-
able that responds to a forcing term. We physical scientists spend our
formative years solving such problems, with the parameters (material
causes) and boundary conditions (formal causes) specified. Apart from
Galileo’s influence [119], this widely-shared early experience may explain
why most of us automatically assume that every natural event can be de-
scribed in terms of efficient causes.

- A final cause requires understanding the intention of some cognitive entity.
  In modern philosophial jargon, this is the “problem of action” which
has recently been analyzed in some detail by philosopher Alicia Juarr-
ero from the perspectives of information theory and nonlinear dynam-
ical systems [420]. Although the reality of intentions has been strongly
rejected by Monod [575], Juarrero has shown how intention can emerge
8.2. OBJECTIONS TO REDUCTIONISM

in a higher-level state space that includes both internal mental states (the cell assemblies of Sec. 7.5.1) and external physical and cultural phenomena.

As Aristotle well knew, several such causes are often involved in a single event, comprising a web rather than a chain; thus we should expect that parameter values, boundary conditions, forcing functions, and cognitive intentions will all combine to influence the outcome of a given dynamical process. What other complications of causality are anticipated?

8.2.3 Nonlinear Causality

In applied mathematics, the term “nonlinear” is usually defined in the context of relationships between efficient causes and their effects. Suppose that a series of experiments on a certain system has shown that cause \( C_1 \) gives rise to effect \( E_1 \); thus

\[
C_1 \rightarrow E_1,
\]

and similarly

\[
C_2 \rightarrow E_2
\]

expresses the relationship between cause \( C_2 \) and effect \( E_2 \). From the discussion in Chap. 1, this relation is linear if

\[
C_1 + C_2 \rightarrow E_{12} = E_1 + E_2.
\]

If, on the other hand, \( E_{12} \) is not equal to \( E_1 + E_2 \), the effect is said to be a nonlinear response to the causes. Equation (8.1) indicates that for a linear system any efficient cause can be arbitrarily divided into components \( (C_1, C_2, \ldots, C_n) \), whereupon the effect will be correspondingly divided into \( (E_1, E_2, \ldots, E_n) \).

Although convenient for analysis,\(^6\) this property is not found in many areas of physical science (as we have seen in Chap. 6) and rarely in the life sciences (see Chap. 7).

Far more common is the nonlinear situation, where the combined effect from two causes is not equal to the sum of their individual effects, and the whole is not equal to the sum of its parts. Nonlinearity is inconvenient for the academic analyst who wishes to publish many papers and advance his career, because multiple causes interact among themselves, allowing possibilities for more outcomes, obscuring relations between cause and effect, and confounding the constructionist. For just this reason, however, nonlinearity plays key roles in the course of biological dynamics.

8.2.4 Time’s Arrow

Our concept of causality is closely connected with our sense of time – thus, the statement “\( C \) causes \( E \)” implies (among other things) that \( E \) does not occur before \( C \) \([119]\) – yet the properties of time may depend on the level of description

\(^6\)Providing a basis for Fourier analysis and Green function methods \([738]\).
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[296, 297], particularly in biological systems [885, 886]. Thus the dynamics underlying molecular vibrations are based on Newton’s laws of motion, in which time is bidirectional. In other words, the direction of time in Newton’s formulation can be changed without altering the qualitative behavior of the system. (The Solar System could as well run backward as forward, as could the pendulum of a clock.) For systems like a nerve axon that consume energy, on the other hand, time has an “arrow;” thus a change in time’s direction makes an unstable nerve impulse stable and vice versa [656]. In Fig. 4.3 under time reversal, for example, the dashed branch would represent stable impulses traveling more slowly than the unstable impulses represented by the solid branch – a striking qualitative difference.

In appealing to Fig. 8.1, therefore, the reductionist must recognize that the nature of the time used in formulating the causal relationship between $C_8$ and $C_8$ may differ from that relating $C_5$ and $C_5$.

8.2.5 Downward Causation

Explanatory reductionists focus their attention on efficient causality, which acts upward through the biological hierarchy [575, 867], possibly explaining a tendency in current science to ignore other types of causality [119]. Importantly, material, formal and final causes can also act downward, because variables at the upper levels of a hierarchy can place constraints (boundary conditions, for example) on the dynamics at lower levels and higher-level intentions can supply material for lower-level activities (think of a runner taking glucose tablets during a marathon). To sort things out, theoretical biologist Claus Emmeche and his colleagues have recently defined three types of downward causation [18, 256].

- **Strong Downward Causation (SDC).** Under SDC, it is supposed that upper-level phenomena act as efficient agents in the dynamics of lower levels. In other words, upper-level organisms can influence or modify the physical and chemical laws governing their molecular constituents, for example by changing the attraction between electric charges. Although New Agers may disagree, there is presently no empirical evidence for the downward action of efficient causation, so SDC is almost universally rejected by life scientists.

- **Weak Downward Causation (WDC).** Rejecting vitalism, WDC assumes that the molecules comprising an organism are governed by some nonlinear dynamics in a phase space, having attractors which include the living organism (see Appendix A). Each of these attractors is endowed with a corresponding basin of attraction, within which the dynamics are stable. Under WDC, an external higher level stimulation might move lower level phase-space variables from one basin of attraction to another.

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7See *What the Bleep!?* at http://www.noetic.org/links/bleep_guide.cfm for a New Age perspective on SDC.
causing a global change in the dynamics of the organism – from fight to flee or feed, for examples. Another example of WDC is as a theoretical explanation for punctuated evolution, whereupon a new species appears suddenly on an evolutionary time scale as the phase point describing a species moves into a new basin of attraction [331, 333, 425].

Because many examples of such nonlinear systems have been studied both experimentally and theoretically, there is little doubt about the scientific credibility of this means for downward causation. As we saw in Sec. 7.2.5, biologists Stuart Kauffman [425] and Brian Goodwin [331] have used a suggestion of mathematician Alan Turing [821] to present detailed discussions of ways that WDC can influence the morphological development and behavior patterns of living organisms.

- **Medium Downward Causation (MDC).** Accepting WDC and again rejecting vitalism, proponents of MDC go further in noting that higher-level dynamics (e.g., the emergence of a higher-level structure) can modify the parameters of an organism’s lower-level phase space through the downward actions of formal and final causes. Examples of MDC include modifications of DNA codes caused by interactions among species under Darwinian evolution and the changes in interneuronal coupling (synaptic strengths) in our brains upon learning.

Evidently both MDC and WDC must be considered in any serious account of Life’s dynamics.

### 8.2.6 Open Systems

In contrast with most formulations of classical physics, living organisms are open systems, requiring an ongoing input of energy and matter (sunlight or food plus oxygen) to maintain their metabolic activities. The steady state\(^8\) of an open system, which was defined and discussed in 1950 by biologist Ludwig von Bertalanffy, corresponds to a stable singular point in an underlying phase space (see Appendix A). Von Bertalanffy considered the corresponding thermodynamics and also defined the important concept of equifinality in which different initial conditions can lead to the same final state (see Chap. 4 and [847]). From the phase-space perspective of Appendix A, equifinality is expected because a solution trajectory can enter the same basin of attraction from several different directions, all of which eventually arrive at the same stable singular point.

Among the biological applications of these concepts mentioned by von Bertalanffy are biophysicist Nicholas Rashevsky’s theoretical cell model [668], morphogenesis (see Sec. 7.2), and nerve impulses (Secs. 4.2 and 7.4 and [737]). Although stationary states of open systems seem strange to physicists, who think in terms of energy-conserving fields, they are well known to electrical engineers, working as they do with nonlinear systems comprising batteries (sources

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\(^8\)The corresponding German term *Fliessgleichgewicht* was coined by von Bertalanffy.
CHAPTER 8. REDUCTIONISM AND LIFE

of energy) and resistors (sinks of energy) in addition to energy-storage elements (inductors and capacitors) [727].

A familiar example of a steady-state solution in an open system is provided by the flame of a candle [268]. Starting with the size and composition of a candle, it is possible to compute the (downward) propagation velocity of the flame \( v \) by equating the rate at which energy is dissipated by the flame (through radiation of light and conduction of heat) to the rate at which energy is released from the wax of the candle [738]. This analysis establishes a rule for finding where the flame will be located at a particular time; thus corresponding to

\[
M_1 \rightarrow M_2
\]

in Fig. 8.1, such a rule is the following: if the flame is at position \( x_1 \) at time \( t_1 \), then it will be at position

\[
x_2 = x_1 + v(t_2 - t_1)
\]

(8.2)

at time \( t_2 > t_1 \), where \( v \) is the velocity of the flame. Because a candle and its flame comprise an open system, however, a corresponding relation

\[
P_1 \rightarrow P_2
\]

cannot be written in terms of the positions and velocities of the constituent atoms and molecules— not even “in principle.” Why not? Because the molecules comprising the physical substrate of the flame are continually changing [75]. Although its velocity is the same, the flame’s heated molecules of air and wax vapor at time \( t_2 \) are different from those at time \( t_1 \). Thus, knowledge of the detailed positions and speeds of the molecules present in the flame at time \( t_1 \) tells nothing about the speeds and positions of the constituent molecules at time \( t_2 \). What remains constant is the flame itself— a higher-level process which can be computed from local averages over its constituents.9

8.2.7 Closed Causal Loops and Networks

In his influential analysis of reductionism, Kim asks (with Aristotle): “How is it possible for the whole to causally affect its constituent parts on which its very existence and nature depend?” [437]. Causal circularity between higher and lower levels is deemed unacceptable because it violates the following “causal-power actuality principle.”

For an object, \( x \), to exercise, at time \( t \), the causal/determinative powers it has in virtue of having property \( P \), \( x \) must already possess \( P \) at \( t \). When \( x \) is being caused to acquire \( P \) at \( t \), it does not already possess \( P \) at \( t \) and is not capable of exercising the causal/determinative powers inherent in \( P \).

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9It might be asserted that “in principle” one could compute the dynamics of all the matter and all the radiation of the universe, but this would require an “omniscient computer,” which is similar to (and perhaps equivalent to) the Calvinist notion of God. True or not, this speculation tells us nothing about reductionism.
8.2. OBJECTIONS TO REDUCTIONISM

There are two objections to this claim, one theoretical and the other empirical. Theoretically, Kim incorrectly supposes that an emergent structure somehow pops into existence at time $t$, which would indeed be surprising. Typically, however, an emergent entity (or coherent structure) begins from an infinitesimal seed (noise) that appears at a lower level of description and develops through a process of exponential growth (instability). Eventually, this growth is limited by nonlinear effects (see Fig. 1.1), and a stable entity is established (the collective research activity in nonlinear science, for example). Think of lighting a bonfire. Upon being barely lit, a small but viable flame grows rapidly before settling down to its natural size for the available fuel.

Thus, using Kim’s notation, both $x$ and $P$ should be viewed as functions of time ($t$), which may be related by nonlinear ODEs as

$$\frac{dx}{dt} = F(x, P),$$
$$\frac{dP}{dt} = G(x, P),$$

where $F$ and $G$ are general nonlinear functions of both $x$ and $P$. Importantly, the emergent structure is not represented by $x(t)$ and $P(t)$ (which are functions of time and can be infinitesimally small), but by $x_0$ and $P_0$, satisfying

$$0 = F(x_0, P_0),$$
$$0 = G(x_0, P_0).$$

Assuming that $x_0$ and $P_0$ are an asymptotically stable solution of this system,

$$x(t) \rightarrow x_0,$$
$$P(t) \rightarrow P_0,$$

as $t \rightarrow \infty$, they establish a dynamic balance between downward and upward causations of the emergent structure (see Appendix A). Thus, Kim’s causal-power actuality principle is a theoretical artifact stemming from his static analysis of a dynamic situation.

Empirically, we have seen in Sec. 6.6.3 that there are many examples of closed causal loops in the feedback mechanisms of mechanical and electrical systems, and the fact that philosophers have been bothered by this concept underscores the fractionation of knowledge prior to the nonlinear-science revolution. In the course of his attempts to realize mathematician David Hilbert’s dream of putting all mathematics on an axiomatic basis, for example, philosopher Bertrand Russell called closed loops of logical implication “vicious circles” and devised various intellectual contortions to deal with this perceived problem, including his theory of types [53]. As mathematicain Kurt Gödel soon

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10 Among the many examples of such problematic loops, I have always been amused by Groucho Marx’s statement: “I wouldn’t want join a club that would have a bum like me as a member.”
showed, however, Hilbert’s procrustean program was doomed to failure and Russell was wasting his considerable intellectual energy [592].

Going back to inventor and engineer James Watt in the eighteenth century, for example, negative feedback (NFB) has been used to govern the speed of engines. Since the 1920s, NFB loops stabilize the performance of electronic amplifiers, making long distance telephone communications possible, and closed causal loops of NFB play key roles in control systems and in mathematician Norbert Wiener’s science of cybernetics (see Sec. 5.2.1 and [874]).11 In biological contexts, NFB plays a role in allosteric regulation of enzyme activity (Sec. 7.1.3 and [892]), and biochemists Manfred Eigen and Peter Schuster suggested over two decades ago that closed causal loops of positive feedback (PFB) around at least three hierarchical levels of dynamic description were necessary for the emergence of living organisms from the the Hadean oceans (Secs. 5.2.2 and 7.6.3 and [240]).

To realize feedback in engineering applications, a signal from one terminal (say the output) is brought back to the input, as shown in Fig. 8.2(a). Here A causes B, which in turn causes A, closing a causal loop and conflating the concepts of cause and effect, to the distress of logically oriented philosophers. If the net gain around the loop exceeds unity, the feedback in positive, and exponential growth of the dynamic variables occurs, whereupon nonlinear effects will eventually limit the growth as a new entity emerges.

11A system with NFB must be carefully designed to avoid unwanted oscillations, which are called “singing” in amplifiers, “hunting” in control systems, and Parkinson’s disease (paralysis agitans) in humans.
8.2. OBJECTIONS TO REDUCTIONISM

A familiar example of a closed causal loop with exponential growth is the flame of a candle that has just barely been lit. The heat from the small flame (A) begins to melt wax (B), which is then drawn into the flame, providing the fuel to make it larger. As the flame becomes larger, yet more heat is produced so more fuel is released, and so on. In other words, A causes B to increase, which in turn causes A to increase, both growing together until they reach the levels of a fully developed flame, the size of which is determined by nonlinear (saturation) effects. The resulting emergent entity is a fully developed flame, and the system is evidently open, because energy is being produced at B and dissipated at A. Thus instead of being logically problematic, closed causal loops are essential elements of nonlinear science.

Nonlinear science offers many examples of PFB and the subsequent emergence of coherent structures. In the physical sciences, structures that emerge from PFB loops include tornadoes, tsunamis, optical solitons, rogue waves, ball lightning (I suppose), and Jupiter’s Great Red Spot, among many others. Biological examples include the nerve impulse, cellular reproduction, flocks of birds and schools of fishes, and the development of new species, in addition to the emergence of Life itself. Psychologically, there are emotions (love, hate, rage, fear, joy, sadness; and so on), recognition of external objects by sound, sight or touch; and the birth of an idea; while in the social sciences, one finds lynch mobs, natural languages, the founding of a new town or city, and the emergence of human culture [151] – to name but a few of the examples that come to mind [741]. The exponential growth stemming from closed loops of PFB is also an essential element of low-dimensional chaos, as we saw in Chap. 2.

In biological and cognitive systems (see Secs. 7.3.5 and 7.5.5), downward causations (WDC and MDC) lead to opportunities for more intricate closed causal networks, which can involve many different levels of both the biological and cognitive hierarchies as is suggested in Fig. 8.2(b). In this diagram, the node A might represent the production of energy within an organism, which induces a muscular contraction B, leading the organism to a source of food C, which is ingested D, helping to restore the original energy expended by A. Additionally, A might energize a thought process E, which recalls a positive memory of taste F, further encouraging ingestion D. The thought E might also induce the generation of a digestive enzyme G which also makes the source of food seem more attractive. Finally, ingestion D might further induce generation of the enzyme G. In this simple example – which is intended only as a cartoon – the network comprises the following closed loops of causation: ABCD, CDG, AEFD, and AEFCG, where the letters correspond to entities at various levels of both the biological and the cognitive hierarchies. With sufficient gain around some of these closed loops, the entire network shown in Fig. 8.2(b) becomes established as a new entity – one’s first experience of eating a Neapolitan pizza, say.

Building on this example, PFB networks can lead to the emergence of entities with seemingly unbounded complexity, relating physiological, mental, intentional and motor levels. Analysis of such networks is not a trivial matter
because the time and space scales for models of living creatures differ by many orders of magnitude as one goes from the biochemical levels of a single cell to the dynamics of a whole organism, and – as molecular biologist David Goodsell has shown in a remarkable little book called *The Machinery of Life* [330] – a single cell is a very complicated dynamical entity. Thus the modeler wonders what can be said about the organization of a living system from a theoretical perspective.

### 8.3 Theories of Life

While much of modern biology is rightly directed toward understanding causal connections between phenotypical (adult) characteristics and DNA codes, common sense suggests that such reductive analyses ought not to comprise it all. Some appropriate fraction of the total research effort should be devoted to understanding the organizational aspects of living creatures, asking questions like: Are biological organisms mechanistic, as Descartes claimed? Is that real duck swimming in the pond over there but an improved version of Jacques de Vaucanson eighteenth-century *canard digérant* (digesting duck) (see Fig. 8.3) which quacked, ate, drank, swam, flapped its wings and defecated? Can we build living machines or must they grow? Is a living organism something more than a mechanism? If so, how does it differ? Can studies of nonlinear science help biologists see the difference?

![I quack, therefore I am!](image)

**Figure 8.3:** Jacques de Vaucanson’s *canard digérant*. (Having the ability to defecate, this mechanical model is in amusing accord with a recent observation that “biology is inherently messy” [429].)

#### 8.3.1 Artificial Life vs. Autopoiesis

Motivated by such questions, the term “artificial life” (ALife) was coined during a workshop organized by computer scientist Chris Langton, sponsored by

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12Descartes never saw Vaucanson’s mechanical duck as it was constructed decades after he died, but he had been youthfully impressed by hydraulic automata at the Palace of Versailles [203].
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the Los Alamos Center for Nonlinear Studies, and held in Los Alamos during September of 1987 [85, 465]. An important cause of this “new science” was the recent growth in computing power (see Fig. 1.3), which became widely evident in the mid-1980s, stimulating the imaginations of computer mavens. In simplest terms, the aim of ALife is to study biological phenomena by seeing how (or if) Life can be simulated on computers; thus this development is a sophisticated outgrowth of mathematician John Conway’s relatively simple “Game of Life” which has fascinated scientists and novices alike since the 1970s [356]. Generally, of course, ALife aims for more advanced computer realizations of previous mechanical models, like the canard digérant shown in Fig. 8.3 and Wiener’s amusing little robot from the 1950s (Sec. 5.2.1). As with Turing’s famous definition of computer intelligence [820], the logic is that if entities are given more and more of the properties of Life, they will eventually become alive.

Among the biological behaviors that have been simulated under ALife are flocking, exploring environments, reproduction, competing for limited resources, and evolution by mutation and selection, but more generally in Langton’s view, ALife studies the synthesis of life-like behavior rather than the analysis of biological organisms and considers “life-as-it-could-be” rather than “life-as-we-know-it.” To these ends, ALife includes proponents of both “weak” and “strong” varieties, with the former group assuming that their field can provide tools to assist the biologists in formulating and testing their theories, and the latter asserting that new life forms will be created within their computers.

Coined in the early 1970s by Chilean biologists Humberto Maturana and Francisco Varela and taken from a Greek word meaning “self-moving,” autopoiesis refers to the ability of a living organism to set its own agenda and move itself, as opposed to the term “allopoietic,” meaning “being moved” [837]. Thus these scientists were thinking of a living cell as an autonomous dynamical system, which is organized as a network of processes (see Fig. 8.2) that continually regenerates its constituent relations [330]. In the words of Maturana and Varela [536]:

An autopoietic machine is a machine organised (defined as a unity) as a network of processes of production (transformation and destruction) of components that produces the components which: (i) through their interactions and transformations continuously regenerate and realise the network of processes (relations) that produced them; and (ii) constitute it (the machine) as a concrete unity in the space in which they (the components) exist by specifying the topological domain of its realisation as such a network.

Thus autopoiesis is a top-down approach toward understanding living systems, proceeding – appropriately, it seems to me – from Life’s empirical properties to its theoretical formulation, as opposed to the bottom-up approach of

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13In July of 2006, interestingly, the term “artificial life” recalled over four million pages on Google.
ALife. By concentrating their attention on Life’s organization, Maturana and Varela have helped to move biological research in an important direction.

8.3.2 Relational Biology

Two key publications that also approach the phenomenon of Life from organizational perspectives are Rashevsky’s seminal paper “Topology and life: In search of general mathematical principles in biology and sociology” (see Sec. 7.2.7 and [667, 668]) and a recent book by Robert Rosen entitled Life Itself: A Comprehensive Inquiry Into the Nature, Origin, and Fabrication of Life. Together with some related essays [690], Rosen’s book presents a coherent account of ideas that began to ferment during his graduate studies on “relational biology” in Rashevsky’s mathematical biophysics program (Sec. 5.2.2), importantly showing that not all natural systems can be simulated [686, 687, 689, 690].

As it is primarily concerned with the organization of living systems, relational biology has an interesting duality with respect to reductionism, which can be oversimplified as follows. Whereas the reductionist would “throw away the organization and keep the underlying matter,” the relational biologist would “throw away the matter and keep the underlying organization” [689]. Of course the reductionist does not entirely ignore the organization as he expects to recapture it upon understanding the dynamics of the material constituents, and the relational biologist does not ignore matter as she expects to realize an interesting organization in some material context, but their starting perspectives and emphases are different.

Under reductionism, organization is merely epiphenomenal, whereas from the relational perspective as Rosen puts it: “the organization of a system has become the main object of study. In essence, organization has become a thing, with its own formal images or models, its own attributes, and its own modes of analysis.” Thus relational biology is in accord with both the spirit and the practice of modern nonlinear science, where mathematically similar dynamics are realized in quite different material contexts, and emergent entities (strange attractors, solitons, nerve impulses, and so on) are viewed as real dynamic objects and given appropriate ontological status.

8.3.3 Mechanisms

As scientific models are often conflated with the underlying reality they strive to represent, Rosen considered these two aspects of an analysis separately, using Fig. 8.4 to emphasize the dichotomous relationship between a natural system ($N$) and a theory ($M$) that has been constructed to model some features of it [689]. The temporal course of causality (1) in the natural system is mimicked more or less accurately by logical or recursive implications (3) in the model. To go from the natural world to initial conditions for the model requires the taking of data (Tycho Brahe’s measurements of positions of the planets, for example, or recordings of spectral peaks by a physical chemist), which Rosen calls “encoding” (2) to the model, and to compare predictions of the model with mea-
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measurements of events in the natural world requires a process of “decoding” (4) from the model (Johannes Kepler’s calculations of elliptical planetary orbits, say, or predictions of molecular resonances).

![Diagram of a scientific model](image)

Figure 8.4: Rosen’s diagram of a scientific model.

Constructing models of natural systems is problematic (how do we know what’s really out there?), but Rosen grounded his work by studying mathematical models of formal systems. Thus the natural system \( \mathcal{N} \) in Fig. 8.4 is replaced by a set \( S \), so his entire formulation remains within the realms of mathematics, wherein he defines two general types of models.

1. **Analytic models**, which are constructed from direct (Cartesian) products of measurements and are tied to the notions of efficient cause and semantic description. Thus each component of an analytic model on a set \( S \) takes the form

\[
M(S) = \prod_{\alpha} f_{\alpha}(S),
\]

a direct product over measurements \( f_{\alpha} \) on the elements of \( S \).

2. **Synthetic models**, which are constructed from direct sums of disjoint subsets of \( S \) and are tied to the notions of material cause and syntactic description. In formulating a synthetic model, the set \( S \) is resolved into a direct sum of subsets \( \{ u_{\alpha} \} \), having corresponding elements \( u_{\alpha} \) on which measurements \( \varphi_{\alpha}(u_{\alpha}) \) are defined, with each \( \varphi_{\alpha} \) an arbitrary function of \( u_{\alpha} \). Then a component for a synthetic model is defined as

\[
P(s) = \sum_{\alpha} r_{\alpha} \varphi_{\alpha}(u_{\alpha}),
\]

a direct sum over measurements \( \varphi_{\alpha} \) on the elements of \( u_{\alpha} \) (where the \( r_{\alpha} \) are numbers). As the \( \{ u_{\alpha} \} \) are the “disjoint elements” assumed under “analysis” in

\[14\]The direct product of two sets \( X \) and \( Y \), denoted \( X \times Y \), is the set of all ordered pairs whose first component is a member of \( X \) and whose second component is a member of \( Y \). Causes are “efficient” because the representation is in a high-level state space, and descriptions are “semantic” because meanings are not excluded from the dynamics at high levels.

\[15\]The direct sum of two disjoint sets \( X \) and \( Y \), denoted \( X + Y \), is the smallest set containing both \( X \) and \( Y \). Causes are “material” because the description is at low levels, and the formulation is “syntactic” because meanings are not defined at low levels.
Sec. 8.1.1, synthetic models are evidently reductive. For examples, the elements of a subset $u_A$ might represent atoms (while the subsets represent molecules) or DNA base pairs (and the subsets represent codons) or neurons (with the subsets representing primary cell assemblies).

Although analytic and synthetic models are usually assumed to be equivalent, Rosen shows they are not necessarily so, claiming this to be "the central question of all theoretical science." He shows that synthetic models can always be formulated as analytic models, but from his proof that direct products are more inclusive than direct sums,\(^\text{16}\) it follows that analytic models cannot in general be formulated as synthetic models. This result is closely related to the facts that mathematical systems cannot in general be reduced to axioms (à la Gödel) (see Sec. 8.3.4 and [592]) and that meanings cannot in general be reduced to grammatical rules in the context of linguistics [690, 758].

Viewing the above discussion of Kim’s analysis from Rosen’s perspective, Fig. 8.1 should be redrawn as in Fig. 8.5, where $M$ is an analytic model (because each component may depend on all aspects of the system) and $P$ is a synthetic model (because it is constructed on disjoint subsets of the fundamental elements of $S$). As $M$ is more general than $P$, Kim’s main conclusion – that physicalism requires explanatory reductionism – is incorrect.

Rosen distinguishes between modeling, which at its best is a creative activity at high levels of description, and simulation, which is based on a low (reductive) level of description and is algorithmic, meaning that it can be done by a computer. For example, the Zeldovich–Frank-Kamenetski (ZF) equation (see Sec. 4.1 and [737, 915])

\[
D \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = \frac{u(u - a)(u - 1)}{\tau},
\]

is a model for both flame-front propagation and nerve-impulse propagation,\(^\text{16}\) To construct this proof, Rosen shows that synthetic models have largest and smallest forms, whereas analytic models do not.
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as is (8.2). Thus with appropriate values of the parameters $D$, $\tau$ and $a\tau$, (8.3) describes the temperature of a candle flame as a function of position ($x$) and time ($t$), which in turn fixes the velocity ($v$) of its propagation in (8.2). Simulation of a candle flame, on the other hand, requires a procedure that computes a solution of fluid equations for the molecules of wax being drawn into the flame, taking account of chemical reactions and the release of energy under combustion. Those systems – like the candle – for which all models – like (8.2) and (8.3) – can be simulated are called mechanisms or simple systems, a classification that includes most of the physical systems described in Chap. 6 and many of the biological systems in Chap. 7.

8.3.4 Complex Systems and Chaotic Emergence

Importantly, Rosen has done for models of the natural world what Gödel did for his science. What was this? In 1931, Gödel startled his colleagues by proving that mathematical systems of modest complexity (number theory, for example) cannot be put on an axiomatic basis, as there will always be true theorems that cannot be proven under a finite set of axioms [592].\footnote{An example of such an unproven theorem in number theory may be Christian Goldbach’s mid-eighteenth conjecture that any even number can be expressed as the sum of two prime numbers.} In other words, there is more to a mathematical system than is captured by its axioms. Let’s see how this unanticipated result translates into natural science.

Rosen called a system that cannot be simulated a complex system. Thus analytic models of complex systems cannot in general be reduced to synthetic models – the defining property of a simple system or mechanism. Physical examples of complex systems include fluid turbulence (shown in Figs. 2.5, 6.12 and 6.13 and discussed in Sec. 6.8.7) and the hierarchy of emergent universes (see Fig. 6.19 in Sec. 6.9.5). Thus although turbulence can be modeled by the Burgers equation

$$D \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = (c + \beta u) \frac{\partial u}{\partial x}$$

(with appropriate choices for the parameters $D$, $c$ and $\beta$), its simulation is not possible, as applied mathematician Horace Lamb noted with amusing despair. Why not? The sizes and shapes of applied mathematician Lewis Richardson’s “big whirls,” “little whirls” and “lesser whirls” in a turbulent fluid cannot be determined by integration of the Navier-Stokes equation equations, because the locations and times at which these myriad new entities begin to emerge from their infinitesimal seeds are not under numerical control. In other words, the emergence of each whirl is not numerically stable, leading to uncontrolled errors (errors that grow exponentially with time) that arise when attempting to compute the turbulent internal geometry of the system. In the context of mental self-organization, philosopher David Newman calls such an uncontrolled process the chaotic emergence of a complex system [599, 600].

Similarly, the heirarchy of universes suggested in Fig. 6.19 is modeled by
the discrete Mandelbrot map

\[ z_{n+1} = z_n^2 + c, \]

where \( z \) is a complex variable and \( c \) is a complex number.\(^\text{18}\) (Recall from Sec. 6.9.5 that Mandelbrot’s iteration starts with \( z_0 = 0 \), and \( c \) is included in the set if \( |z_n| \) remains bounded as \( n \to \infty \).) Although it has not been shown in detail, I expect that a corresponding integration of the Einstein–Hilbert equation (6.15) cannot be carried out because the geometries of my “big black holes,” “little holes” and “lesser holes” would not be under numerical control, leading again to chaotic emergence of a complex system.

Let us next consider some examples of biological systems that have been mentioned in Chap. 7 and in this chapter, asking which are complex and which are merely mechanisms.

**BIOLOGICAL EVOLUTION.** The Darwinian development of life forms over the past four billion years on Earth (see Sec. 7.6) is thought to be chaotically emergent because – as evolutionary biologist Stephen Jay Gould has convincingly argued – the course of “punctuated evolution” would differ greatly on a second run [333, Mayr02]. In other words, the times and locations of the emergence of new species (appearing as abruptly beginning “punctuations” in the fossil record) stem from small seeds (two individuals, say), implying a loss of computational control. It follows that simulations based on lower-level (reductive) descriptions are not feasible because the process is chaotically emergent; thus analytic models cannot be expressed as synthetic models, and the reductive program cannot be carried through.

**COGNITIVE HIERARCHIES.** As noted above, Newman has shown that the development of cognitive structures in a human brain is a chaotically emergent process (see Sec. 7.5.5 and [599, 600]), because identical twins do not develop the same patterns of thought [736]. According to psychologist Donald Hebb, internal hierarchies of cell assembly structures emerge in each person’s had during the course of Life, and these necessarily differ as they appear in response to the happenstance of existence (Sec. 7.5.1 and [366, 367, 368]). For similar reasons, chaotic emergence is expected to characterize the development of natural languages and the growth human cultures [65, 151].

**ARTIFICIAL LIFE.** In general, ALife systems can be either mechanisms or complex systems, depending on the nature of the computations involved [85, 465]. Some ALife models, like Conway’s “Game of Life” [356], are stably simulated on computers; thus they are mechanisms for which analytic models can be reduced to synthetic models, and they merely mimic Life [690]. Other ALife models may be numerically unstable in the sense that two successive computations do not give the same result, indicating that such models are not mechanisms, but this is only a necessary (not a sufficient) condition for being alive.

\(^{18}\)Here the standard terminology is unfortunate. The numbers \( z \) and \( c \) are said to be “complex,” indicating that they are constructed from a real and an imaginary component (i.e., they are two-dimensional vectors). This is unrelated to whether or not the system is “complex” in Rosen’s sense.
In either case, however, the weak ALife program still promises contributions to biological research, which may be its strongest asset.

**PROTEIN FOLDING.** A complicated dynamical system that seems simulatable is a folding protein, upon which biochemists continue to put great effort due to the technical importance of this problem. The crux of protein folding is not numerical instabilities stemming from emergence or chaos, because in principle the computational task is merely to find the configuration (shape) having a minimum of free energy. Unfortunately, however, a typical protein has a very large (nearly immense [255]) number of nearly degenerate (of almost equal energy) conformational states (Sec. 7.2.3 and [298]), which makes the true energy minimum difficult to define and determine. This problem may eventually be overcome with further increases of computing power in machines such as IBM’s “Blue Gene”\(^{19}\) or “Folding@Home” – a cooperative system that employs the spare computing power of many participants.\(^{20}\)

**HUMAN MORPHOLOGIES.** The forms of living organisms are largely determined by their DNA codes, but humans with identical genomes begin to deviate upon aging, because mind-body interactions – stemming from downward acting intentions [420] – induce unpredictable variations in eating patterns, physical activity, use of recreational drugs, and so on. In support of this claim, a recent study of 30 male and 50 female monozygotic (MZ) twins by geneticist Mario Fraga and his colleagues in Spain, Sweden, Denmark, Britain, and the United States shows “widespread ‘epigenetic drift’ associated with aging” (acetylation of histones H3 and H4) [286, 533]. In other words, changes in gene expression were found to occur with aging, which in turn led to differences in the onset of common diseases. For those of my age, a vivid if anecdotal example of epigenetic drift is provided by the saga of Canada’s Dionne quintuplets – five MZ girls born in 1934 who were the subjects of many articles and photo essays during the 1930s and 1940s. To take but one stark measure of their aging, Emilie died in 1954 of an epileptic seizure, Marie in 1970 of a stroke, Yvonne in 2001 of cancer, while Annette and Cecile are still alive as I write in 2006.

**DYNAMICS OF LIFE.** Although the general form of an organism is determined by its genome (the offspring of a duck doesn’t look like a dog), there remains the problem of knowing how it moves and interacts with its environment. As Rosen has demonstrated [689, 690], living organisms are complex systems rather than mechanisms, so analytic models cannot be reduced to synthetic models, and the dynamics of an individual being’s behavior cannot be simulated. While Jacques de Vaucanson’s *canard digérant* (see Fig. 8.3) may have swum like a duck and quacked like a duck, in other words, it did not simulate the behavior of a particular duck.

Whatever the ultimate judgments on the simulatability of these examples, there is now a precise – albeit more restrictive – meaning for the term “complex system” than those that have been loosely employed in academic and general

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\(^{19}\)http://en.wikipedia.org/wiki/Blue\_Gene

\(^{20}\)http://folding.stanford.edu/
discourse. Under this new definition, a complex system can be modeled (of course), but it cannot be simulated from its reductive elements because it combines loops of PFB with downward causation – just the property that leads to chaos and uncontrolled emergence, which in turn are the components of chaotic emergence.

Why should we care about this? Why does it help us to know that biology is complex rather than mechanistic? Perhaps the broadest answer to this question is that one’s thinking becomes released from the shackles of reductionism, in which it is difficult to understand how the constituent atoms (or molecules or DNA codons or neurons or whatever) can have intentions [420]. Thus the concept of a complex system does the work of Descartes’s substance dualism without rejecting physicalism.

On reductionism, interestingly, Rosen concludes as follows [689].

The sequence of state transitions of a system of particles, governed by Newton’s laws of motion, starting from some initial configuration, is then the analog of a theorem in a formalism, generated from an initial proposition (hypothesis) under the influence of the production rules. And just as [proponents of] formalization in mathematics believed that everything could be formalized without loss, so all truth could be recaptured in terms of syntax alone, so particle mechanics came to believe that every material behavior could be, and should be, and indeed must be, reduced to purely syntactical sequences of configurations in an underlying system of particles.

Hence the power of the belief in reductionism, the scientific equivalent of the formalist faith in syntax. Though of course Newtonian mechanics has had to be supplemented and generalized repeatedly, the basic faith in syntax has not changed; indeed, it has been bolstered and made more credible by these very improvements. And there has been no Gödel in physics to challenge that credibility directly. But there is biology.

And there has been a Rosen and there is also nonlinear science.

8.3.5 What Is Life?

This question was famously asked by physicist Erwin Schrödinger in a widely-read little book based on a series of public lectures that he presented at Dublin’s Trinity College in February of 1943 [720]. Following an earlier suggestion of biophysicist Max Delbrück, Schrödinger stated for the first time in clear physical terms that genetic information is embodied in a code of molecular valence bonds, starting physicist Francis Crick and biochemist James Watson on their search for the structure of DNA [579, 720]. For this reason, Schrödinger’s book

\footnote{For example, a “complex system” is defined on Wikipedia as “a system of many parts which are coupled in a nonlinear fashion.”}

\footnote{Delbrück was awarded the 1969 Nobel Prize in medicine for his work on viruses.}
is often cited as support for reductive formulations of biology, but therein he also asked the question: “Is life based on the laws of physics?” to which he replied as follows.

From all we have learned about the structure of living matter, we must be prepared to find it working in a manner that cannot be reduced to the ordinary laws of physics. And that is not on the ground that there is any “new force” or what not, directing the behaviour of the single atoms within the living organism, but because the construction is different from anything we have yet tested in the physical laboratory.

As Schrödinger knew biology in addition to being one of the great physicists of the twentieth century [579], this claim offers little comfort to reductive life scientists, but they seldom mention it. The rest of us, however, may ask where to find these phenomena that “cannot be reduced to the ordinary laws of physics,” and the answer is: within the realms of both biology and nonlinear science. During his war-time tenure as founding director of the Dublin Institute of Advanced Studies, interestingly, Schrödinger tried to move the physics community in the direction of nonlinear science – through the organization of conferences and his personal research on nonlinear electromagnetism as a basis for elementary particle theories (see Sec. 6.1.1) and on Einstein’s gravitational theory (Sec. 6.9) – but he was decades ahead of his time.23

Dedicated to Schrödinger’s nonreductive perspective is a recent book with the same title by evolutionary biologist Lynn Margulis and science writer Dorion Sagan which emphasizes the organizational aspects of living creatures [527]. Following Rashevsky’s relational biology, finally, this question (“What is Life?”) motivated Rosen throughout his decades-long effort to understand and formulate organizational principles for biology [691]. Merely asking the question is significant because this act assumes Life to be noun rather than an adjective (as in: “Is that thing alive?”). By treating “the integrated activities of the organism as a whole” [667], the relational analysis of Rashevsky and Rosen locates Life within organizational structure and focuses attention on a fundamental fallacy of modern biology [689, 690].

Following beliefs of the physics community, many biologists continue to see their field as a special case of reductive physics for which the details are yet to be worked out – thereby avoiding any taint of vitalism – but Schrödinger’s above quote suggests that reductive physics is the special case, neglecting much that falls within the realms of both nonlinear science and biology. Although not identical fields, nonlinear science and biology extensively overlap (as we have seen in the previous chapter), and both are more general than reductive physics because they are not restricted to simple systems (mechanisms, which can be reductively simulated) but include the unbounded and unpredictable phenomena of chaos, emergence, and chaotic emergence as they arise from interactions among the hierarchical levels of complex systems.

23 Walter Moore’s biography gives an excellent survey of Schrödinger’s efforts to promote nonlinear science during his tenure in Dublin [579].
Generalizing the concept of emergence from the nonlinear dynamics at a single level of description to include organizational networks – comprising both downward and upward causations among many levels of a being’s biological, cognitive, and cultural hierarchies as in Fig. 8.2(b) – leads to the very complicated organizational thing, the noun that we call Life. Far greater than the sum of its parts (Death), Life is not a mechanism, like some refined version of de Vaucanson’s duck (Fig. 8.3). Nested within a nonlinear complexity that is unrivaled in the known Universe, Life combines all aspects of a creature’s dynamics: its physiological, conceptual, motivational, and motor activities in an unsimulatable autopoietic network.

Nonlinear science is not yet close to describing this phenomenon, but here, in my view, is where to seek answers to the remaining riddles of biology.
Chapter 9

Epilogue

So much has been done, exclaimed the soul of Frankenstein – more, far more, will I achieve: treading in the steps already marked, I will pioneer a new way, explore unknown powers, and unfold to the world the deepest mysteries of creation.

Mary Wollstonecraft Shelley

In the summer of 1816, young Mary Wollstonecraft Godwin had fled England with her lover, Percy Bysshe Shelley, and their infant son for Geneva, where they mixed with Lord Byron, his mistress (Mary’s step-sister), and John Polidori (his personal physician). As the days were unpleasant, they passed the time at first by reading German ghost stories and then challenged themselves to write tales of terror. Byron responded with a fragment and Polidori came through with a short story that founded modern vampire fiction [654], but the laurel surely went to Shelley née Godwin for her novel Frankenstein, or The Modern Prometheus [762], the plot of which involves the scientific creation of Life – a question that continues to haunt us almost two centuries later.

At the beginning of the nineteenth century, the time was ripe for Shelley’s tale, as Luigi Galvani had recently discovered “animal electricity,” whereby a frog’s leg muscle is caused to twitch through the action of an agent that might or might not be the same as Alessandro Volta’s chemical electricity or Ben Franklin’s atmospheric electricity [729]. Did lightning from the heavens, some wondered, provide the spark of Life for inert matter? By 1850, young Hermann Helmholtz had used Galvani’s biological preparation to show that animal electricity is indeed a unique phenomenon [374], but the surprisingly small speed that he measured was not explained for a century, until the phenomenon of nerve-impulse propagation emerged as an important example of reaction-diffusion propagation within the more general area of nonlinear dynamics.

Growing up in a small New England town during the 1930s and 1940s, I learned of Victor Frankenstein and his desolate creation through the movies – a staple of entertainment in those pre-television days. Although fanciful ver-
visions of her original tale had occasionally appeared in stage plays throughout the nineteenth century, Shelley’s copyright-free concept was enthusiastically taken up by Hollywood’s Universal Studios, which produced a flock of films that liberally mixed Frankenstein and his monster with vampires, other strange creatures, and those infernal hordes of villagers with their wild eyes and dripping torches. Thus emerged a genre that was eventually spoofed by Young Frankenstein, in which Victor’s grandson (played by Gene Wilder in perhaps his greatest screen role) had the benefit of a modern neuroscience education. For an eleven-year-old walking home alone in the dark, however, Mel Brook’s classic sendup was three decades in the future, and there was nothing amusing about Frankenstein Meets the Wolf Man, the dread of which was compounded by the possibility that one or both of this pair might be quietly following me. Such are the images that nest in our brains.

Looking back, it seems oddly significant that I graduated from the university in the same year that physiologists Alan Hodgkin and Andrew Huxley resolved the riddle of nerve impulse propagation, combining experimental physiology with imaginative mathematics in a tour de force that set the stage for the nonlinear-science revolution of the 1970s [390]. Upon joining the academic community, my primary aim was to understand the nature of impulse conduction, which is so central to the movement of our muscles, the reception of signals from our eyes and ears, the formation of neocortical memories, and the beating of our hearts – all stuff that Victor Frankenstein needed to unfold for us “the deepest mysteries of creation.” Motivated by this profound relation to biological dynamics, my professional energies have been devoted to nonlinear science for more than four decades.

So what is the secret of Life? Although rooted in nature, living beings are organized as immensely complex dynamic hierarchies, where “immense” is used in the technical sense proposed by biophysicist Walter Elsasser to denote a finite number of possibilities that is too large to list [255], and “complex” was defined by biomathematician Robert Rosen for the class of natural systems that cannot be reductively modeled [689, 690]. Biological hierarchies achieve their immense complexities through processes of chaotic emergence, a phrase that was coined by philosopher David Newman to describe mental self-organization [599, 600], and can be applied to Darwinian evolution, the growth of biological forms, and their daily dynamics, in addition to some physical examples (fluid turbulence and the hierarchical expansion of our physical Universe). Because they tend to regard biologically oriented physical scientists as academic claim jumpers who willfully ignore the messy features of Life [482], many real biologists remain unaware of these ideas, openly or tacitly assuming reductive perspectives that have been discredited by research in nonlinear science.

Guided by the concepts of immensely complex dynamic hierarchies and of chaotic emergence, however, a fully nonlinear formulation of Life steers deftly between the Scylla of Newtonian physics and the Charybdis of Cartesian dualism, driving a wooden stake through the heart of reductionism while suggesting – pace Jacques Monod – that there may be something to Henri Bergson’s vitalism after all.